

2.1 Derivative Basics: Form F Key

Tutor copy. Every answer was checked by symbolic computation (SymPy, `verify_FJ.py`). All problems are original; provenance is in `PROVENANCE_F.md`. Prepared with assistance from Claude, an AI model made by Anthropic.

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General grading rules

- **Answer-only items** (Q1, Q9): all or nothing, except where a row below says otherwise.
- **Value vs. derivative.** Giving a derivative where a function value was asked (or the reverse), or a number from the wrong column of a table, earns 0 for that part unless a row below says otherwise.
- **Show-work items** (Parts II–IV other than Q9, and Part VI): a bare correct answer earns at most half credit.
- **Definition items** (Q6, Q7): the problem states the required method. A correct answer by differentiation rules earns at most 2 of the 8 points.
- **Method is not graded for efficiency.** A correct answer reached by long but valid work gets full credit; the time it costs is the penalty.

Form F

Item	Answer	Rubric / trap
1(a)	4	<i>Trap:</i> the tangent drawn there invites the slope, 2; earns 0.
1(b)	-2	<i>Trap:</i> giving the height $f(-2) = 4$; earns 0.
1(c)	0	<i>Trap:</i> giving the height 5 of the high point; earns 0.
1(d)	DNE	Corner at $(-1, 1)$: one-sided slopes -4 and 1 . <i>Trap:</i> 0, because $(-1, 1)$ is a low point.
1(e)	$(-4, -3)$ and $(-1, 3)$	3 pts, all or nothing; union notation is fine. Endpoints may not be included. <i>Trap:</i> $(-4, 3)$, which includes the falling stretch $(-3, -1)$ and the corner.
1(f)	$f'(-2) < 0 < f'(-4)$	$f'(-4) = 2$, $f'(-2) = -2$. 3 pts, all or nothing.
2	$y - 5 = 12(x - 2)$, i.e. $y = 12x - 19$	2 pts $p(2) = 5$; 3 pts the equation, any correct form. <i>Trap:</i> the slope is printed in the problem. A student who differentiates p and evaluates at 2 gets 12 again: full credit, but the time is gone. Using 12 as the y -value earns 2 of 5.
3	$y + 4 = 4(x + 1)$, i.e. $y = 4x$	1 pt $f(-1) = -4$; 2 pts $f'(x)$; 1 pt $f'(-1) = 4$; 1 pt equation in any correct form. <i>Trap:</i> $f(-1) = -2 + 1 - 3$ and $f'(x) = 6x^2 + 2x$, $f'(-1) = 6 - 2$: sign slips at a negative input. Writing $x - 1$ for $x - (-1)$ earns 3 of 5.
4(a)	2	3 pts. Simplifies to $y = 2x - 1$. <i>Trap:</i> 5, the coefficient in the familiar slope position; earns 0.
4(b)	1	2 pts. Substitute $x = 1$. <i>Trap:</i> 3, read from the left side; earns 0.
5	6	<i>Trap:</i> q' is printed and invites $q'(-2) = 1$; earns 0.
6	$f'(x) = -\frac{6}{(2x-1)^2}$	2 pts $f(x+h) = \frac{3}{2(x+h)-1} = \frac{3}{2x+2h-1}$, substituted correctly and in parentheses; 2 pts common denominator, numerator $3(2x-1) - 3(2x+2h-1) = -6h$; 2 pts cancel h to get $\frac{-6}{(2x+2h-1)(2x-1)}$; 2 pts let $h \rightarrow 0$. <i>Trap:</i> writing $f(x)+h$ for $f(x+h)$, or losing a sign across the subtraction. Rules only: at most 2.

Item	Answer	Rubric / trap
7	8	1 pt $g(4) = -16$; 1 pt setup; 3 pts factoring out $x-4$ to get x^2-x-4 , by any valid method. Synthetic or long division by $x-4$ works (on 1, -5, 0, 16, with the zero placeholder). So does grouping: $x^3 - 5x^2 + 16 = (x^3 - 4x^2) - (x^2 - 16) = x^2(x-4) - (x-4)(x+4) = (x-4)(x^2-x-4)$. No particular method is required. 3 pts value. <i>Trap</i> : $g(4) = 64 - 80 = -16$, so subtracting it gives +16. A student who divides and drops the $0x$ placeholder gets a wrong quotient. Rules only: at most 2.
8(a)	$-\frac{\sqrt{3}}{2}$	$\frac{1}{2} = \cos \frac{\pi}{3}$, so this is the derivative of $\cos$ at $\frac{\pi}{3}$: $-\sin \frac{\pi}{3} = -\frac{\sqrt{3}}{2}$. <i>Trap</i> : "0/0, DNE" or $+\frac{\sqrt{3}}{2}$; earns 0. 1 pt for recognizing which derivative the limit is (or a correct algebraic setup), 2 pts for the value.
8(b)	32	the derivative of x^4 at 2: $4 \cdot 2^3 = 32$; factoring $x^4-16 = (x-2)(x+2)(x^2+4)$ also works. <i>Trap</i> : 8 (from $4 \cdot 2$), or "0/0"; earns 0. 1 pt for recognizing which derivative the limit is (or a correct algebraic setup), 2 pts for the value.
9	(a) 0 (b) 0	3 each. <i>Traps</i> : (a) $-\sqrt{3} \sin(\pi/6) = -\frac{\sqrt{3}}{2}$; (b) $f'(-1) = 5$. In (b), $m(x)$ is the same number for every x , so $m'(x) = 0$; the problem says "for every x ".
10(a)	$2x^{-2/3} + 10x^{-3}$	1 pt rewriting $6x^{1/3} - 5x^{-2}$; 2 pts derivative. <i>Trap</i> : $-5 \cdot (-2) = +10$; $\frac{1}{3} - 1 = -\frac{2}{3}$.
10(b)	$\frac{2}{3}x$	1 pt rewriting $\frac{1}{3}x^2 - 2$ (for $x \neq 0$); 2 pts derivative. <i>Trap</i> : dividing only the first term by $3x$; differentiating top and bottom separately earns 0.
10(c)	$2x + 6 - 4 \cos x$	1 pt rewriting $x^2 + 6x + 9 - 4 \sin x$; 2 pts derivative. <i>Trap</i> : $(x+3)^2 \neq x^2 + 9$.
10(d)	$2 \sin x$	1 pt rewriting nothing; π^2 is a constant; 2 pts derivative. <i>Trap</i> : writing 2π for the derivative of π^2 ; the derivative of $-2 \cos x$ is $+2 \sin x$.
11	$2\sqrt{3} - \frac{\pi}{3}$	3 pts $f'(x) = 4 \cos x - 2x$; 2 pts value. <i>Trap</i> : $\cos \frac{\pi}{6} = \frac{\sqrt{3}}{2}$, not $\frac{1}{2}$; using $\frac{1}{2}$ gives $2 - \frac{\pi}{3}$, which earns 3.
12	$x = \frac{7\pi}{4}$	2 pts $f'(x) = \sqrt{2} + 2 \sin x$; 2 pts $\sin x = -\frac{\sqrt{2}}{2}$; 2 pts the solutions in the interval. <i>Trap</i> : $\frac{5\pi}{4}$ also solves $\sin x = -\frac{\sqrt{2}}{2}$ but lies outside $(\frac{3\pi}{2}, 2\pi]$; listing it costs 2.
13	III	6 pts, all or nothing. In III the dashed curve is $f(x) = \frac{x^3}{2} - \frac{3x}{2}$ and the solid curve is $f'(x) = \frac{3x^2}{2} - \frac{3}{2}$: it is zero exactly where the dashed curve turns ($x = \pm 1$), positive where that curve rises, and negative where it falls. Wrong choices: I: shifted: the zeros of the other curve are at $x = 0$ and $x = 2$, not at the turning points $x = \pm 1$; II: the other curve is zero where the cubic crosses the x -axis ($x = \pm\sqrt{3}$), not where it turns; IV: right zeros, wrong sign: it is negative where the cubic rises. <i>Trap</i> : assuming the dashed curve is always the derivative; the styles swap between graphs.
14(a)	0	$h'(x) = 3f'(x) - 2g'(x) + 2x$, so $h'(2) = 3(-2) - 2(-1) + 4$. 1 pt rule, 2 pts correct table entries, 1 pt value. <i>Trap</i> : the table lists $f'(x)$ first and $f(x)$ third. Using the value columns gives $3(1) - 2(4) + 4 = -1$, which is $h(2)$; earns 1 (for the rule, if shown). Forgetting that x^2 contributes $2x = 4$ gives -4.
14(b)	21	$f(3) = 5$ is a number, so $k(x) = 5x^2 - g(x)$ and $k'(x) = 10x - g'(x)$; $k'(2) = 20 - (-1) = 21$. 2 pts $k'(x)$, 2 pts value. <i>Trap</i> : using $f'(3) = -2$ gives -7; earns at most 1.
14(c)	$x = 3$ only	$(f+g)' = f' + g'$, which is 5, 3, -3, 0 at $x = -1, 0, 2, 3$. 2 pts for using $f' + g'$, 2 pts for the answer. <i>Trap</i> : $x = 0$, where $f(x) + g(x) = 0$: there the graph crosses the x -axis; it is not flat. Earns 0.

Points: $14 + 5 + 5 + 5 + 2 + 8 + 8 + 6 + 6 + 12 + 5 + 6 + 6 + 12 = 100$.